

绝对值不等式

1. $|x^2 - 5x + 5| < 1$

$\Leftrightarrow -1 < x^2 - 5x + 5 < 1$

$\Leftrightarrow \begin{cases} x^2 - 5x + 6 > 0 \\ x^2 - 5x + 4 < 0 \end{cases}$

$\Leftrightarrow \begin{cases} (x-2)(x-3) > 0 \\ (x-1)(x-4) < 0 \end{cases}$

$\Leftrightarrow \begin{cases} x < 2 \text{ or } x > 3 \\ 1 < x < 4 \end{cases}$

$\Leftrightarrow 1 < x < 2 \text{ or } 3 < x < 4$

2. $1 < |x| < 3$

$\Leftrightarrow 1 < x < 3 \text{ or } -3 < x < -1$

3. $4 < |2x-3| \leq 7$

$\Leftrightarrow 4 < 2x-3 \leq 7 \text{ or } -7 \leq 2x-3 < -4$

$\Leftrightarrow \frac{7}{2} < x \leq 5 \text{ or } -2 \leq x < \frac{1}{2}$

4. $|x-2| < |x+1|$

$\Leftrightarrow (x-2)^2 < (x+1)^2$

$\Leftrightarrow x^2 - 4x + 4 < x^2 + 2x + 1$

$\Leftrightarrow 3 < 6x$

$\Leftrightarrow x > \frac{1}{2}$

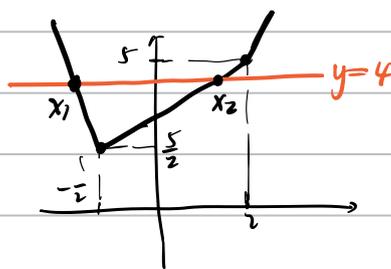
5. $|2x+1| + |x-2| > 4$

解: 记 $f(x) = |2x+1| + |x-2|$

分段点为 $-\frac{1}{2}$ 和 2

$f(-\frac{1}{2}) = \frac{5}{2}$ $f(2) = 5$

图像如下:



当 $-\frac{1}{2} < x < 2$ 时, $h(x) = 2x+1+2-x = x+3$

$x+3 = 4 \Rightarrow x_2 = 1$

当 $x < -\frac{1}{2}$ 时, $h(x) = -2x-1+2-x = -3x+1$

$-3x_1+1 = 4 \Rightarrow x_1 = -1$

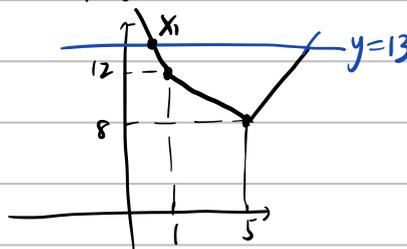
$\therefore x < -1 \text{ or } x > 1$

6. $2|x-1| + 3|x-5| < 13$

解: 记左端 = $f(x)$, 分段点为 $1, 5$.

$f(1) = 12$, $f(5) = 8$

图像如下:



当 $x < 1$ 时, $f(x) = 2(1-x) + 3(5-x) = -5x + 17 = 13$

$\Rightarrow x_1 = \frac{4}{5}$

当 $x > 5$ 时, $f(x) = 5x - 17 = 13$

$\Rightarrow x_2 = 6$

$\therefore \frac{4}{5} < x < 6$

7. $|x+2| - 2|x-1| > 3$

解: 记 $f(x) = |x+2| - 2|x-1|$, 分段点为 $(-2, 1)$

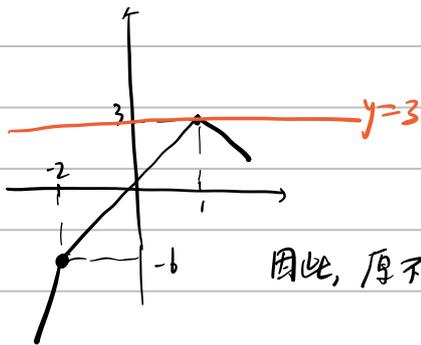
$x < -2$ 时, $f(x) = -x-2-2(1-x) = -x-2-2+2x = x-4$

$x > 1$ 时, $f(x) = 4-x$

$f(-2) = -6$

$f(1) = 3$

图象如下:



8. $x-1 \leq |x^2+x+1|$

解: $\because x^2+x+1 = (x+\frac{1}{2})^2 + \frac{3}{4} > 0$ 恒成立

(或用 $\Delta = 1-4 < 0$ 判断)

$\therefore x-1 \leq x^2+x+1$

$\Leftrightarrow x^2+2 \geq 0$

$\Leftrightarrow x \in \mathbb{R}$