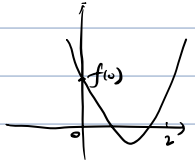
二次方程：根的分布问题

1. $x^2 - mx + m = 0$

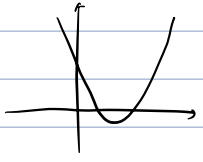
(1) $\begin{cases} 0 < x_1 < 2 \\ 0 < x_2 < 2 \end{cases}$



$$\begin{cases} f(0) = m > 0 \\ 0 < \frac{m}{2} < 2 \\ \Delta = m^2 - 4m > 0 \\ f(2) = 4 - m > 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} m > 0 \\ 0 < m < 4 \\ m < 0 \text{ or } m > 4 \\ m < 4 \end{cases} \Leftrightarrow \text{无解}$$

(2) $\begin{cases} x_1 > 0 \\ x_2 > 0 \end{cases}$

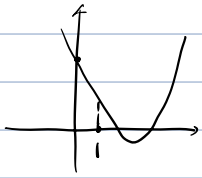


$$\begin{cases} f(0) = m > 0 \\ \frac{m}{2} > 0 \\ \Delta = m^2 - 4m > 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} m > 0 \\ m < 0 \text{ or } m > 4 \end{cases}$$

$$\Leftrightarrow m > 4$$

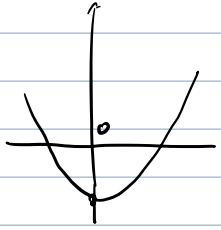
(3) $\begin{cases} x_1 > 1 \\ x_2 > 1 \end{cases}$



$$\begin{cases} f(1) = m > 0 \\ \frac{m}{2} > 1 \\ \Delta = m^2 - 4m > 0 \\ f(1) > 0 \end{cases}$$

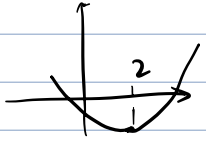
$$\Leftrightarrow \begin{cases} m > 0 \\ m > 2 \\ m < 0 \text{ 或 } m > 4 \\ 1 > 0 \end{cases} \Leftrightarrow m > 4$$

(4) $\begin{cases} x_1 > 0 \\ x_2 < 0 \end{cases}$



$\Leftrightarrow f(0) < 0 \Leftrightarrow m < 0$

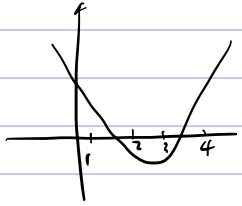
(5) $\begin{cases} x_1 < 2 \\ x_2 > 2 \end{cases}$



$\Leftrightarrow f(2) < 0 \Leftrightarrow 4 - m < 0$

$\Leftrightarrow m > 4$

(b) $\begin{cases} 1<x_1<2 \\ 3<x_2<4 \end{cases}$



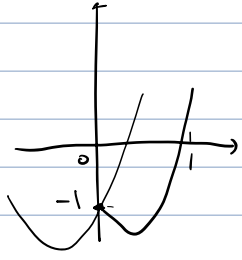
$\Leftrightarrow \begin{cases} f(1)>0 \\ f(2)<0 \\ f(3)<0 \\ f(4)>0 \end{cases} \Leftrightarrow \begin{cases} 1>0 \\ 4-m<0 \\ 9-2m<0 \\ 16-3m>0 \end{cases}$

$\Leftrightarrow \begin{cases} m>4 \\ m>\frac{9}{2} \\ m<\frac{16}{3} \end{cases} \Leftrightarrow \frac{9}{2}<m<\frac{16}{3}$

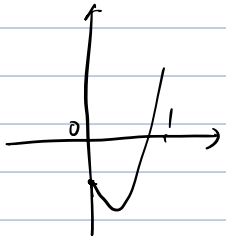
2. $2x^2-mx-1=0$

(1) $0<x<1 \Leftrightarrow f(1)=2-m-1=1-m>0$

$\Leftrightarrow m<1$



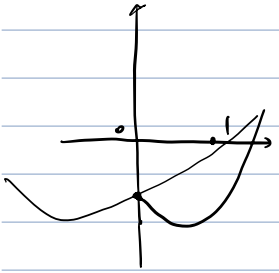
(2) $\begin{cases} 0<x_1<1 \\ 0<x_2<1 \end{cases}$ 不可能



(3) $(0,1)$上没有根 $\Leftrightarrow f(1)<0$

$\Leftrightarrow 2-m-1<0$

$\Leftrightarrow m>1$

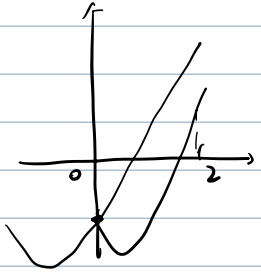


(4) $(0,2)$上只有1个实根 $\Leftrightarrow f(2)>0$

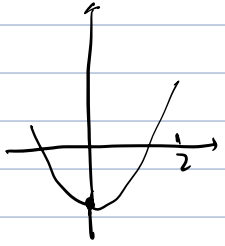
$\Leftrightarrow 8-2m-1>0$

$\Leftrightarrow 2m<7$

$\Leftrightarrow m<\frac{7}{2}$



(5) $(0,2)$上有2个不等根　不可能

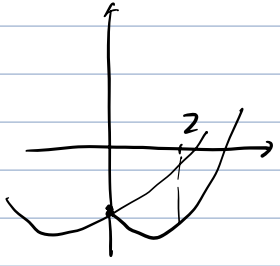


(6) $(0,2)$上没有实根 $\Leftrightarrow f(2)<0$

$\Leftrightarrow 8-2m-1<0$

$\Leftrightarrow 2m>7$

$\Leftrightarrow m>\frac{7}{2}$



3. $f(x)=(a-1)x^2+4x-1$ 在$(-1,1)$内恰有一个零点

解：①若 $a-1=0$ 即 $a=1$，$f(x)=4x-1=0 \Rightarrow x=\frac{1}{4}\in(-1,1)$

②若 $a-1\neq 0$ 即 $a\neq 1$

则 $f(-1)f(1)<0 \Leftrightarrow (a-1-4-1)(a-1+4-1)<0$

$\Leftrightarrow (a-6)(a+2)<0$

$\Leftrightarrow -2<a<6$

综上，$-2<a<6$