

等腰三角形的多解性问题

1. 解: (1) 腰为3, 底为5, $\because 3+3 > 5$, \therefore 可能

此时, $C = 3+3+5 = 11$

(2) 腰为5, 底为3, $\because 3+5 > 5$, \therefore 可能

此时, $C = 5+5+3 = 13$

综上, 周长 $C = 11$ 或 13

2. 解: (1) 腰为3, 底为7, $\because 3+3 < 7$ \therefore 不可能

(2) 腰为7, 底为3, $\because 3+7 > 7$ \therefore 可能

综上, 周长 $C = 3+7+7 = 17$

3. 解: (1) 腰为4, 则底为 $9 - 4 \times 2 = 1$

$\because 1+4 > 4$ \therefore 可能

(2) 底为4, 则腰为 $\frac{9-4}{2} = 2.5$

$\because 2.5+2.5 > 4$ \therefore 可能

综上, 腰为4或2.5

4. 解: 设腰为 x , 底为 y . 则

$$2x + y = 16$$

$$y = 16 - 2x$$

$$\begin{cases} x=1 \\ y=14 \end{cases} \because 1+1 < 14 \therefore \text{不可能}$$

$$\begin{cases} x=2 \\ y=12 \end{cases} \because 2+2 < 12 \therefore \text{不可能}$$

$$\begin{cases} x=3 \\ y=10 \end{cases} \because 3+3 < 10 \therefore \text{不可能}$$

$$\begin{cases} x=4 \\ y=8 \end{cases} \because 4+4 = 8 \therefore \text{不可能}$$

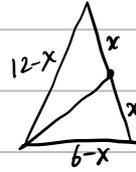
$$\begin{cases} x=5 \\ y=6 \end{cases} \because 5+5 > 6 \therefore \text{可能}$$

$$\begin{cases} x=6 \\ y=4 \end{cases} \because 4+6 > 6 \therefore \text{可能}$$

$$\begin{cases} x=7 \\ y=2 \end{cases} \because 2+7 > 7 \therefore \text{可能}$$

综上, 腰长为5或6或7

5. 解: (1)



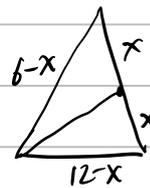
此时, $2x = 12 - x$

$$3x = 12$$

$$x = 4$$

腰长为8, 底为2, $\because 2+8 > 8$, \therefore 可能

(2)



此时, $2x = 6 - x$

$$3x = 6$$

$$x = 2$$

腰长为4, 底为10 $\because 4+4 < 10$ \therefore 不可能

综上, 腰长为8

6. 解: (1) 顶角为 80°

(2) 底角为 80° , 此时, 顶角为

$$180^\circ - 80^\circ \times 2 = 20^\circ$$

7. 解: (1) 若 $\angle A = \angle B = 40^\circ$, 则

$$\angle C = 180^\circ - 40^\circ \times 2 = 100^\circ$$

$$\angle C \text{ 的外角为 } 180^\circ - \angle C = 80^\circ$$

(2) 若 $\angle C = 40^\circ$

$$\text{则 } \angle C \text{ 的外角为 } 180^\circ - \angle C = 140^\circ$$

综上, $\angle C$ 的外角为 80° 或 140°

7. 解法二 [用外角定理]

$\angle C$ 的外角 = 与 $\angle C$ 不相邻的内角之和.

(1) 若 $\angle A = \angle B = 40^\circ$

则 $\angle C$ 外角 = $\angle A + \angle B = 80^\circ$

(2) 若 $\angle C = 40^\circ$

则 $\angle C$ 外角 = $180^\circ - 40^\circ = 140^\circ$

8. 解 (1) 设内角分别为 $x^\circ, x^\circ, 4x^\circ$

则 $x + x + 4x = 180$

$\Rightarrow 6x = 180$

$\Rightarrow x = 30$

此时, 底角为 30° , 顶角为 120°

(2) 设内角分别为 $x^\circ, 4x^\circ, 4x^\circ$.

则 $x + 4x + 4x = 180$

$\Rightarrow 9x = 180$

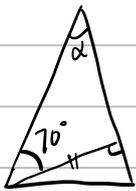
$\Rightarrow x = 20$

此时, 底角为 80° , 顶角为 20°

综上, 顶角为 120° 或 20°

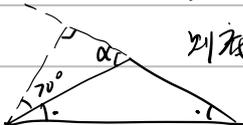
9. 解: (1) $\alpha = 90^\circ - 70^\circ = 20^\circ$

则底角为 $\frac{180^\circ - \alpha}{2} = \frac{180^\circ - 20^\circ}{2} = 80^\circ$



(2) $\alpha = 90^\circ - 70^\circ = 20^\circ$

则底角为 $\frac{\alpha}{2} = 10^\circ$

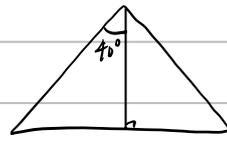


综上, 底角为 80° 或 10°

10. 解: (1) 若底边上的高与腰夹角为 40° , 则

顶角 = $40^\circ \times 2 = 80^\circ$

底角 = $90^\circ - 40^\circ = 50^\circ$



(2) 若腰上的高和另一腰夹角为 40° , 则

顶角 = $90^\circ - 40^\circ = 50^\circ$

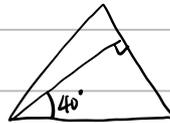
底角 = $\frac{180^\circ - 50^\circ}{2} = 65^\circ$



(3) 若腰上的高和底边夹角为 40° , 则

底角 = $90^\circ - 40^\circ = 50^\circ$

顶角 = $180^\circ - 50^\circ \times 2 = 80^\circ$

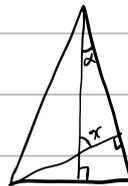


综上, 三个内角分别为 $80^\circ, 50^\circ, 50^\circ$,

或 $50^\circ, 65^\circ, 65^\circ$

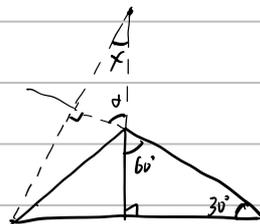
11. 解: (1) 若顶角为 30° , 则 $\alpha = \frac{30^\circ}{2} = 15^\circ$

$x = 90^\circ - \alpha = 90^\circ - 15^\circ = 75^\circ$



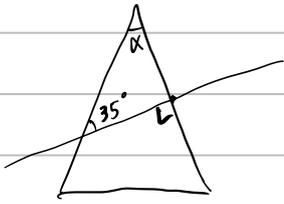
(2) 若底角为 30° , 则 $\alpha = 60^\circ$

$x = 90^\circ - 60^\circ = 30^\circ$

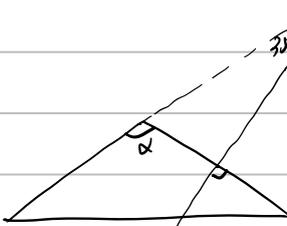


综上, 所求角为 75° 或 30°

12. 解: (1) 若顶角为锐角, 则 $\alpha = 90^\circ - 35^\circ = 55^\circ$



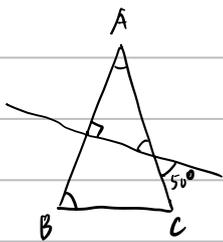
(2) 若顶角为钝角, 则 $\alpha = 35^\circ + 90^\circ = 125^\circ$



综上, 顶角为 55° 或 125°

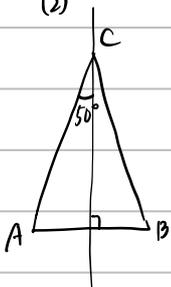
13. (1) $\angle A = 90^\circ - 50^\circ = 40^\circ$

$$\angle B = \frac{180^\circ - \angle A}{2} = \frac{180^\circ - 40^\circ}{2} = 70^\circ$$



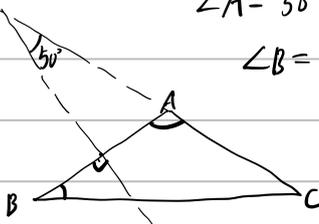
(2) 顶角 = $50^\circ \times 2 = 100^\circ$

$$\angle B = \frac{180^\circ - 100^\circ}{2} = 40^\circ$$



(3) $\angle A = 50^\circ + 90^\circ = 140^\circ$

$$\angle B = \frac{180^\circ - \angle A}{2} = \frac{180^\circ - 140^\circ}{2} = 20^\circ$$



综上, $\angle B = 70^\circ$ 或 40° 或 20°